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ON ONE CASE OF INTERACTION OF STATIONARY PLANE OBLIQUE SHOCK WAVES IN A POLYTROPIC GAS

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The interaction of plane oblique shock waves, one of which overtakes the other, at an arbitrary angle in a polytropic gas with a constant polytropic index has been considered in the context of the hydrodynamic theory of shock waves. The pattern of the decay of a discontinuity arising in this case and the dependence of its dynamics on the angle of interaction of the waves have been investigated on the basis of numerical calculations. The characteristic angles at which a shock-wave configuration develops and their dependence on the velocity of initial shock waves have been determined.

Let a stationary plane shock wave (SW1) propagate with a velocity D_a in an inhomogeneous isotropic space filled with a polytropic gas at rest (we will assume for definiteness that this gas is air with a polytropic index $k = 1.4$ and a density $\rho_0 = 1.29$ kg/m^3 at an initial pressure $p_0 = 10^5$ Pa). The front of this wave (line AOB in Fig. 1) divides the indicated space into two (upper and lower) semispaces. The parameter of the gas downstream of the SW1 (upper semispace in Fig. 1) can be determined using the system of equations [1]

$$p_1 = \frac{2\rho_0 D_a^2}{k+1} - \frac{k-1}{k+1} p_0 , \quad K_1 = \frac{k - 1 + (k+1)\dfrac{p_1}{p_0}}{k + 1 + (k-1)\dfrac{p_1}{p_0}} , \quad \rho_1 = K_1 \rho_0 , \quad U_1 = \frac{D_a}{K_1} , \tag{1}$$

where U_1 is the velocity of the gas downstream of the SW1 front (directed perpendicularly to the wave front, Fig. 1). If the SW1 front is brought into coincidence with a coordinate system, the total pattern will be as follows: the initial gas upstream of the SW1 front (lower semispace in Fig. 1) moves to this front with a velocity D_a, while the velocity of the gas downstream of the wave front is equal to U_1 and its pressure and density are equal to ρ_1 and p_1. Let another stationary plane shock wave (SW2 in Fig. 1) begin to propagate with a velocity D_b in the upper semispace at an angle φ to the SW1 plane. From the viewpoint of a stationary observer, this corresponds to the case where SW1 propagating with a velocity D_a in a gas is caught up by an another stationary plane shock wave (SW2) propagating with a velocity $D = \sqrt{D_a^2 + D_b^2 - 2D_a D_b \cos(\pi - \varphi)}$ at an angle φ to the SW1. Let us bring the point of contact of SW1 with SW2 (point 0 in Fig. 1) into coincidence with the origin of a coordinate system. The velocity of the gas flow (in the plane of the diagram in Fig. 1) along the line AB coincident with the position of the SW1 front is determined by the ratio $q = D_b/\sin\varphi$; however, the total velocity of the gas propagating under AB (i.e., upstream of the SW1 front) will be equal to $q_0 = \sqrt{q^2 + D_a^2}$ and the flow will propagate at an angle

$$\beta = \arcsin \frac{1}{\sqrt{1 + \left(\dfrac{q}{D_a}\right)^2}} \tag{2}$$

to the horizontal. The velocity of the gas flow downstream of SW1 (propagating above AB, region "1" in Fig. 1) will be equal to $q_1 = \sqrt{q^2 + U_1^2}$ in this coordinate system and the flow will propagate at an angle

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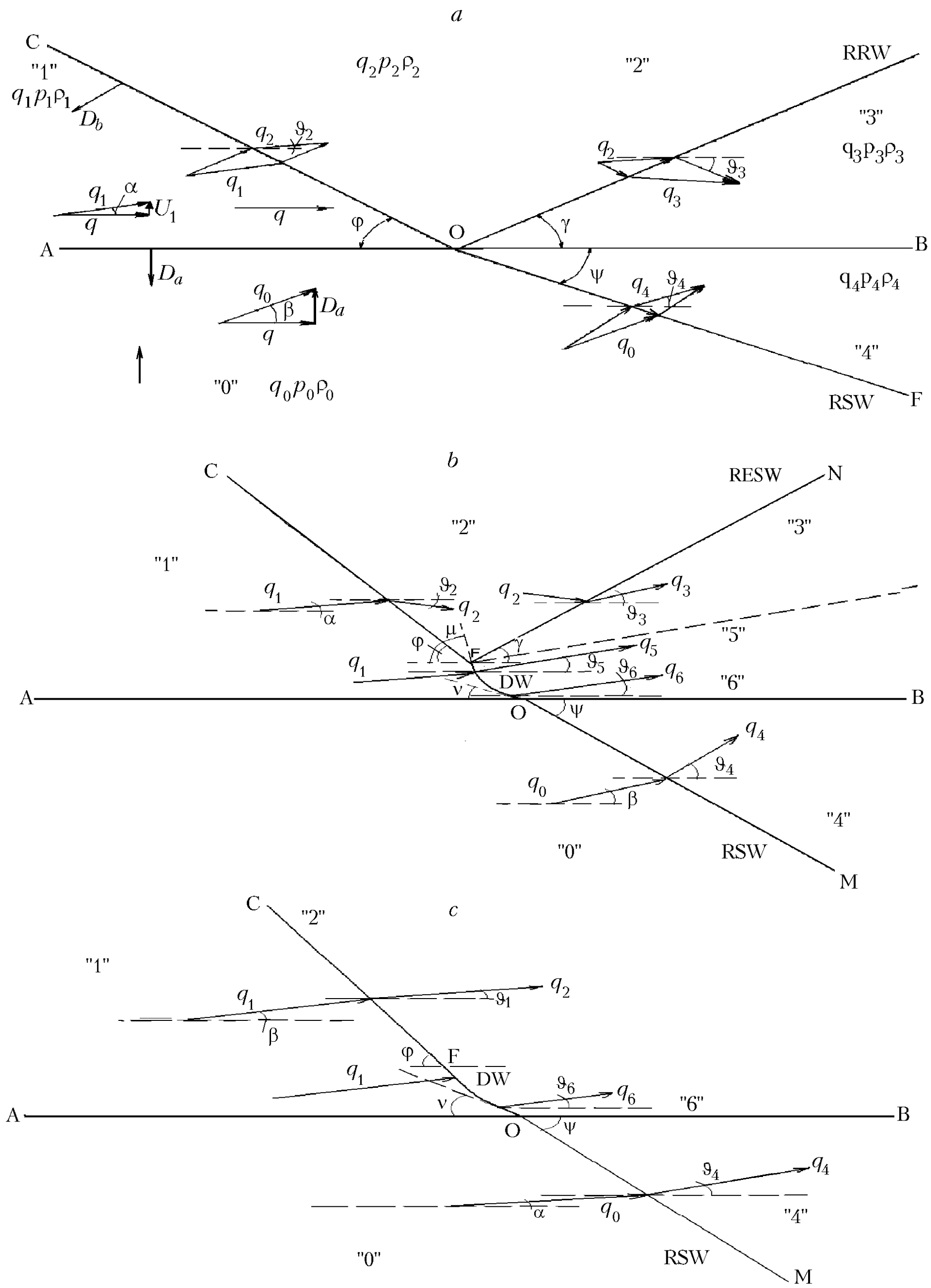

Fig. 1. Diagrammatic representation of shock-wave configurations formed as a result of a regular interaction (a), a strong irregular shock interaction (b), and a weak irregular shock-free interaction (c). RSW – refracted shock wave; RESW – reflected shock wave; RRW – reflected rare-faction wave; DW – distorted wave.

$$\alpha = \arcsin \frac{1}{\sqrt{1 + \left(\frac{q}{U_1}\right)^2}}. \tag{3}$$

The parameters of the gas downstream of the SW2 front (region "2" in Fig. 1) can be determined from the system of equations [1]

$$p_2 = \frac{2\rho_1 q_1^2}{k+1}\sin^2(\alpha+\varphi) - \frac{k-1}{k+1}p_1\,,\quad K_2 = \frac{k-1+(k+1)\dfrac{p_2}{p_1}}{k+1+(k-1)\dfrac{p_2}{p_1}}\,,\quad \rho_2 = K_2\rho_1\,,$$

$$q_2 = q_1\cos(\alpha+\varphi)\sqrt{1+\frac{\tan^2(\alpha+\varphi)}{K_2^2}}\,,\quad \Omega = \arccos\frac{1}{\sqrt{1+\dfrac{\tan^2(\alpha+\varphi)}{K_2^2}}}\,,\quad \vartheta_2 = \begin{cases}\Omega-\varphi\,, & \Omega\ge\varphi\,;\\ \varphi-\Omega\,, & \varphi>\Omega\,.\end{cases} \tag{4}$$

A refracted shock wave (RSW) begins to propagate from the space under the line AOB in Fig. 1, downstream the front of which (region "4" in Fig. 1)

$$p_4 = \frac{2\rho_0 q_0^2}{k+1}\sin^2(\beta+\psi) - \frac{k-1}{k+1}p_0\,,\quad K_4 = \frac{k-1+(k+1)\dfrac{p_4}{p_0}}{k+1+(k-1)\dfrac{p_4}{p_0}}\,,\quad \rho_4 = K_4\rho_0\,,$$

$$q_4 = q_0\cos(\beta+\psi)\sqrt{1+\tan^2(\beta+\psi)\,K_4^{-2}}\,,\quad \omega = \arccos\frac{1}{\sqrt{1+\tan^2(\beta+\psi)\,K_4^{-2}}}\,,\quad \vartheta_4 = \begin{cases}\omega-\psi\,, & \omega\ge\psi\,;\\ \psi-\omega\,, & \psi>\omega\,.\end{cases} \tag{5}$$

Then a discontinuity begins to propagate and, in doing so, restores the balance of the gas located to the right of point O and above the line AOB. Since the pressure p_2 of the gas downstream of SW2 exceeds, even at very small angles φ, the pressure p_4 downstream of the refracted shock wave, this discontinuity should be a rarefaction wave, which as is known [2], is weak and propagates with a velocity of sound c_2. Thus, the angle γ formed by the front of this wave with the SW1 front (line AOB in Fig. 1) will be equal to

$$\gamma = \vartheta_2 + \arcsin\frac{c_2}{q_2}\,, \tag{6}$$

where

$$c_2 = \sqrt{\frac{kp_2}{\rho_2}}\,. \tag{7}$$

The parameters of the gas downstream of the front of the reflected rarefaction wave (RRW) are determined from the system of equations (region "3" in Fig. 1a and b)

$$U_3 = c_2 + \frac{p_2-p_3}{\rho_2 c_2}\,,\quad K_3 = 1+\frac{p_2-p_3}{\rho_2 c_2^2}\,,\quad U_3 = K_3 c_2\,,\quad \rho_3 = \frac{\rho_2}{K_3}\,,$$

$$q_3 = \sqrt{q_2^2\cos^2(\gamma-\vartheta_2)+K_3^2c_2^2}\,,\quad \vartheta_3 = \gamma - \arccos\frac{1}{\sqrt{1+K_3^2\dfrac{c_2^2}{q_2^2\cos^2(\gamma-\vartheta_2)}}}\,. \tag{8}$$

The parameters of all flows formed in different regions of the space considered can be determined from the systems of equations (1)–(8). The eighteen equations (1)–(8), except for additional equations, are insufficient for calculating

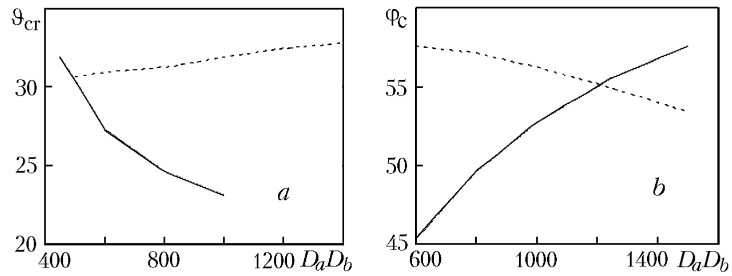

Fig. 2. Dependence of the critical angle of the regular interaction ϑ_{cr} (a) and the angle of change of the supersonic regime of the flow downstream of the SW2 front for a subsonic regime φ_c (b) on the initial velocities of the SW1 D_a and SW2 D_b. ϑ_{cr}, φ_c, deg.

the 20 variables q, q_0, q_1, q_2, q_3, q_4, p_1, p_2, p_3, p_4, α, β, γ, θ_2, θ_3, θ_4, ρ_1, ρ_2, ρ_3, and ρ_4. The total system of equations (1)–(8) is closed by the conditions at the interface into which portion OB (Fig. 1) of the SW1 front is transformed:

$$p_3 = p_4 \,, \quad q_3 \sin \vartheta_3 = q_4 \sin \vartheta_4 \,. \tag{9}$$

Condition (9) provides a balance at the interface between two flows (in the semispace consisting of regions "1", "2", and "3" and in the semisphere comprising regions "0" and "4"). Numerical solution of the combined system of equations (1)–(8) at condition (9) allows one to determine the parameters of all flows formed as a result of the above-described discontinuity decay. This system can be solved as long as angle φ is $\sim\pi/6$. This critical (for the configuration considered) angle will be denoted by ϑ_{cr}; the dependence of this angle on the velocities of the SW1 and SW2 is illustrated in Fig. 2a. It is seen that ϑ_{cr} increases with increasing D_b and, on the contrary, decreases with increasing D_a. This angle changes within the narrow range from 32 to 28^{o}. Figure 3a shows the dependences of the characteristic angles of the shock-wave configuration (reflected-rarefaction-wave angle γ and refracted-shock-wave angle ψ) on the initial angle formed by the fronts of the SW1 and SW2 (φ). When the increasing angle φ reaches ϑ_{cr}, the character of the flow changes grossly. As in the case of interaction of oblique shock waves with the interface, point O is positioned above the interface (SW1 front in our case); in Fig. 1b and c, to this point corresponds point F. In a space, these points represent straight lines perpendicular to the plane of a diagram. These lines are connected by a distorted shock wave (DSW), the front of which is perpendicular to curve FO in the plane of the diagram in Fig. 1b. The parameters of the gas downstream of the distorted-wave front change smoothly and do not disturb the balance established. At the newly formed triple point F, only the solution with a reflected shock wave, represented graphically in Fig. 1b, is stable (solution with a rarefaction is unstable). The problem on the parameters of the flow near this point can be solved using the system of equations (1)–(5), supplemented, for region "3," with the equations

$$p_3 = \frac{2\rho_2 q_2^2}{k+1} \sin^2(\gamma - \vartheta_2) - \frac{k-1}{k+1} p_2 \,, \quad K_3 = \frac{k - 1 + (k+1)\dfrac{p_3}{p_2}}{k + 1 + (k-1)\dfrac{p_3}{p_2}} \,, \quad \rho_3 = K_3 \rho_2 \,,$$

$$q_3 = q_2 \cos(\gamma - \vartheta_2) \sqrt{1 + \tan^2(\gamma - \vartheta_2) K_3^{-2}} \,, \quad \eta = \arccos \sqrt{\frac{1}{1 + \tan^2(\gamma - \vartheta_2) K_3^{-2}}} \,, \quad \vartheta_3 = \begin{cases} \gamma - \eta \,, & \gamma \ge \eta \,; \\ \eta - \gamma \,, & \gamma < \eta \,. \end{cases} \tag{10}$$

Moreover, the system of equations (5), determining the parameters of region "4," should be transformed for region "5":

$$p_5 = \frac{2\rho_1 q_1^2}{k+1} \sin^2(\beta + \mu) - \frac{k-1}{k+1} p_1 \,, \quad K_5 = \frac{k - 1 + (k+1)\dfrac{p_5}{p_1}}{k + 1 + (k-1)\dfrac{p_5}{p_1}} \,, \quad \rho_5 = K_5 \rho_1 \,,$$

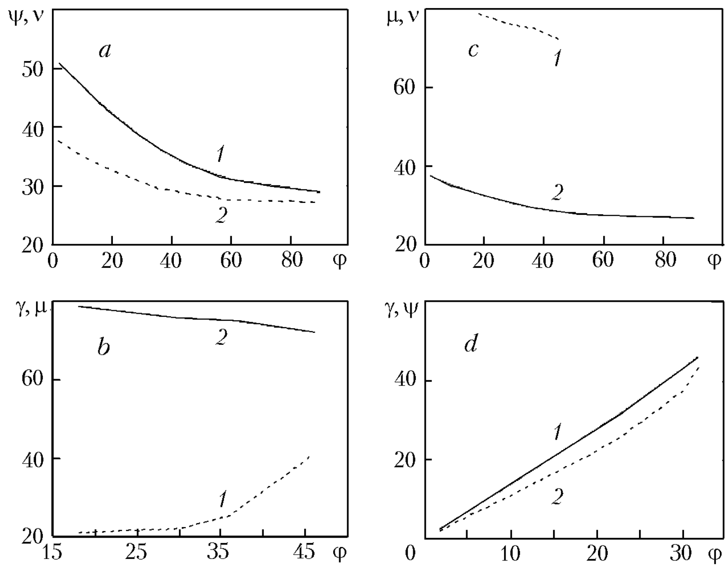

Fig. 3. Dependence of the determining angles of a shock-wave configuration: ψ (1) and ν (2) (a), γ (1) and μ (2) (b), μ (1) and ν (2) (c), γ (1) and ψ (2) (d) on the angle φ between the SW1 and SW2. All the angles are given in deg.

$$q_5 = q_1 \cos(\beta+\mu)\sqrt{1+\tan^2(\beta+\mu)\,K_5^{-2}}\,, \quad \xi = \arccos\sqrt{\frac{1}{1+\tan^2(\beta+\mu)\,K_5^{-2}}}\,, \quad \vartheta_5 = \begin{cases} \xi-\mu\,, & \xi<\mu\,; \\ \mu-\xi, & \mu\le\xi\,. \end{cases} \tag{11}$$

Here, μ is the angle of entry of the distorted wave to the triple point F. The last desired angles of the configuration formed — the angle of entry of the distorted wave to the contact point O (Fig. 1b) ν and the angle ψ formed by the refracted shock wave with the reflected wave — will be determined by solving Eqs. (5) in combination with the equations

$$p_6 = \frac{2\rho_1 q_1^2}{k+1}\sin^2(\beta+\nu) - \frac{k-1}{k+1}p_1\,, \quad K_6 = \frac{k-1+(k+1)\dfrac{p_6}{p_1}}{k+1+(k-1)\dfrac{p_6}{p_1}}\,, \quad \rho_6 = K_6\rho_1\,,$$

$$q_6 = q_1 \cos(\beta+\nu)\sqrt{1+\tan^2(\beta+\nu)\,K_6^{-2}}\,, \quad \sigma = \arccos\sqrt{\frac{1}{1+\tan^2(\beta+\nu)\,K_6^{-2}}}\,, \quad \vartheta_6 = \begin{cases} \sigma-\nu\,, & \nu<\sigma\,; \\ \nu-\sigma, & \nu\ge\sigma\,. \end{cases} \tag{12}$$

The combined system of equations (1)–(4) and (10)–(12) is closed by the conditions on lines OB and FN (Fig. 1b)

$$p_3 = p_5\,, \quad q_3\sin\vartheta_3 = q_5\sin\vartheta_5\,; \tag{13}$$

$$p_4 = p_6\,, \quad q_4\sin\vartheta_4 = q_6\sin\vartheta_6 \tag{14}$$

and includes all relations necessary for calculating the parameters of the flow formed as a result of the discontinuity decay described. Figure 3b–d presents the dependences of the angles γ, ψ, μ, and ν on the increase in the angle of contact of the SW1 with the SW2 φ. Note that, in the case considered, the angle μ at the triple point is always larger than the angle ν at the contact point (Fig. 3c). This means that the convex part of the distorted wave will be presented, in this case, to the incident flow. If the plane projection of a distorted wave on the plane of the diagram can be represented, in the first approximation, as a circular arc, this wave will have the form shown in Fig. 1b. Further numerical investigation has shown that such a configuration, which will be called the strong irregular shock interaction, exists as long as the angle φ reaches a value at which the flow downstream of the SW2 front becomes subsonic. This angle was determined for the first time in [3] and was denoted there by φ_c. The dependence of φ_c on D_a and D_b is shown in Fig. 2b. It is seen that φ_c decreases with increase in D_b and increases with increase in D_a; however, this angle remains fairly large (45–60^o) and exceeds the value of ϑ_{cr}. In all the calculations done, we complied with this rule.

When φ begins to exceed φ_c, the configuration changes once again because the reflected shock wave disappears and the SW2 front is transformed directly (without formation of the triple point) into a distorted wave (Fig. 1c) and the angle μ becomes equal to the angle φ. This configuration is called the strong irregular shock-free interaction. Its calculation is most simple — it is necessary to solve the combined system of equations (5), (12) at condition (14). Shock-wave configurations arising as a result of such an interaction of shock waves, called by the author the overtaking interaction, were numerically investigated in detail only for air. Nonetheless, calculations performed for other gases (helium, neon, krypton, nitrogen, and methane) allow the conclusion that the order in which the configuration is transformed and its general form will be the same for any polytropic gas. Since the hydrodynamic-theory relations for shock waves are fairly general in character, it may be suggested that the evolution of a shock-wave configuration with change in the angle of interaction will proceed in a similar manner for many materials differing substantially in their properties from polytropic gases.

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NOTATION

c, velocity of sound, m/sec; D, velocity of a shock wave, m/sec; k, polytropic index; p, pressure, Pa; q, total velocity of a gas flow, m/sec; U , gas-flow velocity projection perpendicular to the corresponding shock wave, m/sec; α and β, angles of deviation of the total-flow-velocity vector from the horizontal in regions "0" and "1," deg; γ, angle formed by a reflected wave with the horizontal, deg; ϑ, angle of deviation of the total-flow-velocity vector from the horizontal in regions "2"–"6" in dependence on the index, deg; μ and ν, angles of entry of the distorted wave to the triple point and the contact point respectively, deg; ρ, density, kg/m^3; φ, angle formed by the front of the SW1 and SW2 with the horizontal, deg; ψ, angle formed by the refracted shock wave with the horizontal, deg. Subscripts: 0, 1, 2, 3, 4, 5, and 6, denote the relation of quantities to the corresponding regions of the space; a and b, denote the relation of quantities to the SW1 and SW2; c, change of the supersonic regime to a subsonic regime; cr, critical value of a parameter.

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